TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 5 2020

Scientific Calculator Assumed

Rates of Change and Differential Equations

STUDENT'S NAME _____

DATE: Monday 24 August **TIME:** 50 minutes **MARKS**: 47

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: **Three Scientific Calculators**, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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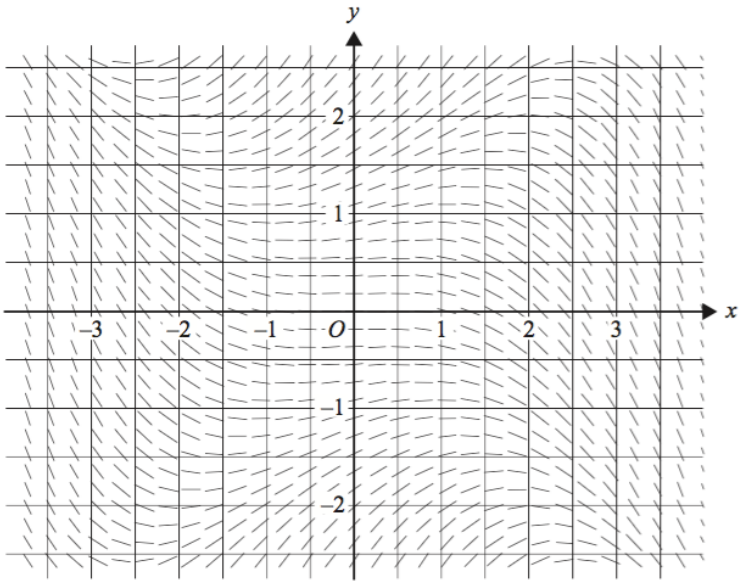
1. (3 marks)



The direction field for a certain differential equation is shown above.

(a) Sketch the solution curve to the differential equation that passes through the point $(-2.5,1)$. [2]

(b) Which of the following points does it pass through? [1]

A. (0, 2) **B.** (1, 1) **C.** (3, –1) **D.** (3, –0.5) **E.** (–0.5, 2)

2. (5 marks)

For the differential equation $\frac{dy}{dx} = \frac{1+y^2}{2xy}$, solve for y in terms of x, given that when $x = 1,\ y = -1$.

3. (11 marks)

A small particle, P, describes simple harmonic motion along a straight with centre O. Two points, A and B, lie on this straight line with A between O and B such that $OA = 3\,m$ and $AB = 1\,m$. At A the speed of the particle is 32 ms^{-1} and at B its speed is 24 ms^{-1}.

(a) Using the equation $v^2 = k^2(A^2 - x^2)$, determine the value of A and k. [4]

(b) Determine the period of the motion [2]

(c) Determine the maximum speed of P [2]

(d) Determine the time to travel from A to B [3]

4. (8 marks)

Audrey's activity is to ride a mini speedboat. To stop at the correct boat dock, she needs to stop the engine and allow the boat to be slowed down by air and water resistance. At time t seconds after the engine has been stopped, the acceleration of the boat, a ms^{-2}, is related to its velocity, v ms^{-1}, by

$$a = -\frac{1}{10}\sqrt{196 - v^2}$$ (you may need the integral $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$)

Audrey stops the engine when the speedboat is travelling at 7 metres per second.

(a) Determine an equation for velocity in terms of time. [3]

(b) Determine the time it takes for the speedboat to come to rest. Give your answer in seconds in terms of π. [2]

(c) Calculate the distance it takes the speedboat to come to rest, from when the engine is stopped. Give your answer in metres correct to one decimal place. [3]

5. (10 marks)

The population of a culture is represented by the equation $N(t) = \dfrac{20}{1+10e^{-\frac{t}{100}}}$, where N is the number of individuals (in thousands) at any time t hours.

(a) When will the population reach 5000? [3]

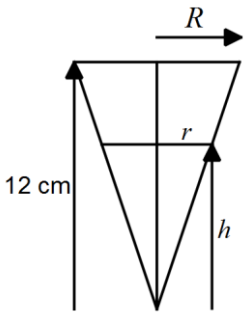
(b) Show that the rate of growth $\dfrac{dN}{dt} = kN(20-N)$ and determine the value of the constant k. All working must be shown in order to receive full marks. [4]

(c) Given that the population after 5 hours is approximately 1903, calculate the approximate increase in the population during the following 5 hours using the incremental formula. Give your answer to the nearest integer. [3]

6. (10 marks)

The height of a conical glass is 12 cm with a base radius R cm. Water is being poured in at a constant rate of k cm^3 min^{-1} and the glass is filled in 2 minutes. (see diagram).

(a) Determine $\frac{dh}{dt}$ when the glass is filled to one-half of its height? [5]



(b) Show that $\frac{dh}{dt} = 2^{\frac{5}{3}}$ cm min^{-1} when half of the water has been poured in? [5]